Abatement technology and the environment-growth nexus with education

Xavier Pautrel*†

forthcoming in Environmental and Resource Economics

Abstract

The present paper adopts a Yaari (1965)–Blanchard (1985) overlapping generations model with Lucas (1988)'s human capital accumulation in order to examine how environmental taxation influences growth. It challenges conventional result that tighter environmental tax has either no long term effect at all, or has a positive long term effect. In the case of Cobb-Douglas production functions and logarithmic utilities, this paper demonstrates that the technology used in the abatement sector determines the existence and direction of the growth effect. In the case of output taxation, if the abatement sector is relatively more (*respectively less*) intensive in human capital than the final output sector, tighter environmental tax increases (*resp. reduces*) growth in the long term. This result always holds true for finite life expectancy but for infinite life expectancy it only holds true when the labour supply is endogenous.

The transitional impact of tighter environmental policy is also investigated. We find that the magnitude of the short term effects of environmental taxation varies according to the technology used in the abatement and final output sectors, and that there is a reverse trade-off between the short and the long term effects.

Keywords: Growth; Environment; Overlapping generations; Human capital; Abatement.

*Université de Nantes, Laboratoire d'Économie et de Management de Nantes (LEMNA), Institut d'Économie et de Management de Nantes - IAE, Chemin de la Censive du Tertre, BP 52231, 44322 Nantes Cedex 3, France. *Mail*: xavier.pautrel@univ-nantes.fr

†I thank the co-editor Michael Rauscher and two anonymous referees for their remarks. The usual disclaimer applies.

1 Introduction

Although the role of human capital accumulation on economic growth has been substantially investigated, the effect of environmental taxation on education and growth has not received the same level of attention at a theoretical level. Moreover, the findings of the few previous papers of this topic tend to agree that environmental tax has no long term influence on human capital accumulation when the final output is the source of pollution (Gradus and Smulders, 1993, Hettich, 1998). The present paper re-examines this finding, both in the long and the short term, by examining the impact of environmental taxation on growth in a Lucas (1988) model with endogenous labour. In contrast to approaches prevalent to the existing literature *(i)* it relaxes the basic assumption of similar production technologies in the final output and abatement sectors, and *(ii)* extends the analysis of the environment-growth nexus with education to agents with finite lives.

This paper is motivated by the fact that most theoretical papers on the environment-growth relationship assume that abatement services are produced in a similar way to final production. However, abatement activities are actually produced in a sizable proportion of specialised industries, termed collectively the "*eco-industry*", which provide environmental goods and services to polluters.[1] Therefore, abatement technology differs from output production technology. The stimulation of abatement activity by environmental policy can provoke sizable re-allocation effects which in turn modify how environmental taxation influences growth, both in the short and the long term. Hence, it is important to investigate whether the simplifying assumption of the existence of similar technologies in the final goods and in the abatement sectors affects the environment–growth nexus.

The second motivation lies in the work of Pautrel (2009, 2012a), who demonstrated that taking into account finitely-lived agents (in the Yaari (1965)-Blanchard (1985) perpetual youth model) introduces new channels for transmitting environmental policy to economic growth. The appearance of a new generation at each date generates a generational turnover that disconnects the aggregate consumption growth from the interest

[1]The Organization for Economic Cooperation and Development and the Statistical Office of the European Commission (OECD/Eurostat, 1999, p.9), define eco-industry as the set of "*(...) activities which produce goods and services to measure, prevent, limit, minimise or correct environmental damage to water, air, and soil, as well as problems related to waste, noise and eco-systems. These include cleaner technologies, products and services which reduce environmental risk and minimize pollution and resource use.*" Nimubona and Sinclair-Desgagné (2011) reported that the eco-industry represents 1.7% of employment and 2.2% of GDP in the European Union and the United States and that it is expected to generate revenue of US$ 800 billion by 2015, making it similar in size to the aerospace and pharmaceutical sectors (see Sinclair-Desgagne, 2008, for more data).

rate. It leads environmental tax to positively influence long term growth, under the human capital accumulation growth model (Pautrel, 2012a) and reduces its negative impact under the AK growth model (Pautrel, 2009). These results call for an extension of the analysis of the environment–growth nexus to finitely lived agents, even in the simple structure of the Yaari (1965)-Blanchard (1985) model.

This paper is in line with the seminal contributions of Gradus and Smulders (1993), Van Ewijk and Van Wijnbergen (1995), Hettich (1998) and with the theoretical contributions in which the influence of environmental tax on growth was recently investigated (Grimaud and Tournemaine, 2007, Pautrel, 2008, 2009, 2012a,b).[2] Gradus and Smulders (1993) and Van Ewijk and Van Wijnbergen (1995) were among the first authors to investigate how the environment influences human capital accumulation. Under the framework of a Lucas (1988) model in which agents are infinitely living, these contributions demonstrated that if educational activities are not directly affected by pollution, environmental tax can never influence human capital accumulation in the long term. Gradus and Smulders (1993) assumed that pollution arises from physical capital and depreciates the stock of human capital, while Van Ewijk and Van Wijnbergen (1995) suggested that output is the source of pollution and that pollution depreciates the ability of agents to learn. Hettich (1998) challenged these results, however, by introducing endogenous labour into the Lucas (1988) framework and by assuming no direct impact of pollution on human capital accumulation. Under these assumptions, he found that environmental tax promotes education and growth, through the channel of labour supply. Because abatement is produced with the same technology as output, the final goods market equilibrium implies that final output is used for abatement activities and household consumption. Because tighter environmental tax compels firms to increase their abatement activities, final output net of abatement reduces and the rise in environmental tax is made at the expense of household consumption. Households therefore substitute leisure for education to counteract this negative effect, pollution decreases through the substitution of physical capital (the pollutant factor) for human capital (the non-pollutant factor), and the growth rate rises. However, Hettich (1998) demonstrated that this result only holds true when pollution arises from physical capital. When the final output is the source of pollution by contrast, tighter environmental policy reduces both the returns to physical capital and the wage rate, which is an element of the returns to education. Even if an environmental tax reduces the ratio of physical to human capital, this does

[2] Note that in the present paper we do not address how pollution affects health and life expectancy.

not reduce pollution, because both factors pollute. Only costly abatement activities can reduce it. Because the decrease in the ratio of physical to human capital is exactly offset by the increased environmental tax, the long term growth rate thus remains unaffected.

Grimaud and Tournemaine (2007) used an R&D model to demonstrate that tighter environmental policy promotes growth when education directly enters the utility function as a consumption good. These authors departed from the basic structure of assuming the existence of a similar technology between the output sector and abatement sector by modelling an R&D sector that aimed to create knowledge in order to reduce the flow of pollution emissions. Higher environmental tax increases the price of the good whose production pollutes and therefore the relative cost of education diminishes. Agents thus increase their investment in human capital accumulation. Further, because education is the engine of growth, growth rises in the steady state. As highlighted by the authors, the way in which education influences utility is crucial to their results, while the modelling of abatement technology is important, too.

In the present paper, we re-examine the environment–growth nexus with education when environmental taxation is applied to final output and abatement services, relaxing the basic assumption of the presence of similar technologies in the final output and abatement sectors[3] and assuming that life expectancy is finite. We use the Yaari (1965)-Blanchard (1985) overlapping generations model with Lucas (1988)'s human capital accumulation and environmental concerns, logarithmic utility and Cobb–Douglas technologies. Under these assumptions, we study both the long term and the short term outcomes of tighter environmental tax and investigate the case where physical capital income is taxed.

The contributions of this paper are threefold. First, we demonstrate that the assumptions made on abatement technology influence how environmental tax affects growth in the long and short term. In the realistic case, where the final output and abatement sectors differ in terms of factor intensities and when final output is taxed, *(i)* environmental tax applied to final output influences long term human capital accumulation and *(ii)* the relative factor intensity determines whether this impact is favourable or unfavourable. Environmental tax boosts (*respectively harms*) long term human capital accumulation when the abatement sector is relatively more intensive in human capital (*resp. phys-*

[3]For the sake of simplicity, we only take into account the differences in relative factor intensity in the final output and abatement sectors. More complex and more realistic modelling of abatement activities is beyond the scope of this paper.

ical capital) compared with the output sector.[4] This result follows from the fact that tighter environmental tax increases abatement activities and generates factor reallocations between sectors. Because factorial intensities differ across sectors, this leads to a relative scarcity of the factor that is intensively used in the abatement sector. When the abatement sector is relatively more intensive in human capital (*resp. physical capital*), the reward of human capital relative to that of physical capital rises (*resp. diminishes*). Agents are thus incited to invest more (*resp. less*) in education and human capital accumulation increases (*resp. decreases*). Our finding always holds true for finite life expectancy, but for infinite life expectancy it only holds true when the labour supply is endogenous. Therefore, the result found by Hettich (1998) when pollution arises from final output is a special case that relies on the assumption of similar technologies in the final output and abatement sectors.

Moreover, our numerical simulations of the transitional dynamics show that when considering how environmental taxation influences growth, there is a trade-off between the short and long term[5] if the abatement sector is more intensive in human capital than the final output sector. Therefore, the growth rate decreases in the short term while it increases in the long term with respect to its pre-tax level. Our results thus suggest that the structure of the economy at the sectorial level could crucially affect how the long and short term impacts of the environmental taxation influence growth. Because in developed countries it seems to be relevant that the abatement sector is relatively intensive in human capital, our results also show that particular attention should be paid to the short term effects of environmental taxation in these countries.

Our second contribution refers to the case of similar technology and finite life expectancy. First, we demonstrate that environmental tax has no long term growth effect when it is applied to national income. The generational turnover effect introduced by finite-life expectancy is therefore insufficient to influence positively the environmental tax-growth relationship by itself. This finding conflicts with the result obtained by Pautrel (2012a) who showed that the generational turnover effect leads environmental policy to promote human capital in the long term when pollution comes from physical capital. This finding arises for the same reason as shown by Hettich (1998), namely that abatement and final goods are produced using the same technology. Second, we demonstrate that the "*labour supply mechanism*" found by Hettich (1998) with infinitely-lived agents

[4]In the following analysis, we consider that the most relevant case for developed countries is that abatement sectors are more intensive in human capital.

[5]This term is borrowed from the anonymous referee who highlighted this point to me.

no longer applies when a finite lifetime is taken into account.

Finally, we investigate whether our results still hold true when environmental taxation is applied to physical capital income rather than national income. We demonstrate that environmental policy always promotes long term human capital accumulation whatever the technologies used in the final output and abatement sectors. As previously mentioned, when the abatement sector is relatively more intensive in physical capital, the reward of physical capital relative to that of human capital increases. Nevertheless, compared with the case where output is taxed, this increase is lower because environmental policy diminishes only the interest rate and not the wage rate. Therefore the downward pressure of environmental tax on the interest rate always compensates for the aforementioned increase in the relative reward of physical capital, indicating that the global impact is higher human capital accumulation.

The remainder of the present paper is structured as follows. Section 2 presents the basic framework of the model. Section 3 investigates the long-term influence of environmental tax on the economy and its impact during transition. Section 4 examines the case where environmental tax is applied to physical capital income. Section 5 concludes.

2 The general framework

2.1 Household behaviour

As noted in the introduction, we use the Yaari (1965)-Blanchard (1985) overlapping generations model with human capital accumulation and environmental concerns. Time is continuous. Each individual born at time s faces a constant probability of death per unit of time $\beta \geq 0$. Consequently his or her life expectancy is $1/\beta$. When β increases, life span decreases. Fertility is at the replacement level such that the number of births equals the number of deaths at any time.[6] At time s, a cohort of size β is born. At time $t \geq s$, the cohort born at s has a size equal to $N(s,t) = \beta e^{-\beta(t-s)}$ and the constant population is equal to $N(t) = \int_{-\infty}^{t} N(s,t)ds = 1$. There are insurance companies and there is no bequest motive.

The expected utility function of an agent born at $s \leq t$ is

$$\int_{s}^{\infty} [\log c(s,t) + \xi_l \log l(s,t) - \zeta \log S(t)] \, e^{-(\varrho+\beta)(t-s)} dt \qquad (1)$$

[6]We assumed a constant population for simplicity. Considering population growth would not modify our qualitative results. Proof available upon request

where $c(s,t)$ denotes the consumption in period t of an agent born at time s, $\varrho \geq 0$ is the rate of time preference, $S(t)$ is the stock of pollution at date t and $l(s,t)$ is the leisure time at date t of an agent born at date $s \leq t$. The exogenous parameters $\xi_l > 0$ and $\zeta > 0$ measure the weight in utility attached to leisure and pollution, respectively.

The representative agent can increase his or her stock of human capital by devoting time to schooling, according to the Lucas (1988)'s model. Because each agent allocates a proportion $u(s,t) \in]0,1[$ of his or her time to production and a proportion $l(s,t) \in]0,1[$ to leisure, his or her remaining time for education is $1 - u(s,t) - l(s,t)$. The evolution of the individual stock of human capital is thus

$$\dot{h}(s,t) = B\left[1 - u(s,t) - l(s,t)\right] h(s,t) \tag{2}$$

with $\dot{h}(s,t) \equiv dh(s,t)/dt$. Parameter B is the efficiency of schooling activities and $h(s,t)$ is the stock of human capital at time t of an individual born at time s. For analytical convenience, we assume that the human capital of the agent when he or she is born, $h(s,s)$, is inherited from the dying generation. To capture the intergenerational transmission of knowledge, we follow the assumption of Bovenberg and Van Ewijk (1997) by considering that newborns inherit the average aggregate human capital stock from the dying generation, that is $h(s,s) = H(s)$ (population being equal to unity).[7]

Households face the following budget constraint:

$$\dot{a}(s,t) = \left[r_n(t) + \beta\right] a(s,t) + u(s,t)h(s,t)w(t) - c(s,t) \tag{3}$$

where $a(s,t)$ is the financial wealth in period t, $w(t)$ represents the wage rate per effective unit of human capital $u(s,t)h(s,t)$ and r_n is the after-tax interest rate.[8] Following Yaari (1965) and Blanchard (1985), we assume the existence of a perfectly competitive life insurance sector that offers actuarially fair annuity contracts. As a result of the foregoing, the annuity rate of interest includes an annuity payment equal to $\beta a(s,t)$.[9] In addition to the budget constraint, a transversality condition must be satisfied in order to prevent

[7]Assuming that $h(t,t) = \iota H(t)$ with $\iota \in]0,1[$ would not modify our qualitative results. Proof available upon request.

[8]We introduce the after-tax interest rate r_n because in section 4 we examine the case where environmental tax is applied to physical capital income. For the moment there is no tax on physical capital income and therefore $r_n = r$ represents the real interest rate.

[9]Each agent concludes a contract with an insurance company that pays him or her an annuity proportional to his or her wealth during life. In exchange, agents transfer their overall wealth on the day they die (see Blanchard and Fisher, 1989, pp. 116-117). Assuming imperfect annuity markets (like Heijdra and Mierau, 2010) would not modify the qualitative results of our model. Proof available upon request.

households from accumulating debt indefinitely:

$$\lim_{v\to\infty}\left[a(s,v)e^{-\int_t^v (r_n(z)+\beta)dz}\right]=0$$

The representative agent chooses the time path for $c(s,t)$, his or her working time $u(s,t)$ and his or her leisure time $l(s,t)$, by maximising (1) subject to (2) and (3). This yields (see Appendix A)

$$\dot{c}(s,t)=\left[r_n(t)-\varrho\right]c(s,t) \tag{4}$$

Integrating (3) and (4) and combining the results provides the consumption at time t of an agent born at time s:

$$c(s,t)=(\varrho+\beta)\left[a(s,t)+\omega(s,t)\right]$$

where $\omega(s,t)\equiv\int_t^\infty\left[u(s,\nu)h(s,\nu)w(\nu)\right]e^{-\int_t^\nu[r_n(\zeta)+\beta]d\zeta}d\nu$ is the present value of lifetime earning.

Utility-maximization also provides the following two relationships (see Appendix A):

$$l(s,t)=\xi_l\frac{c(s,t)}{w(t)\ h(s,t)} \tag{5}$$

and

$$\frac{\dot{w}(t)}{w(t)}+B[1-l(s,t)]=r_n(t)+\beta \tag{6}$$

Equation (5) is individual leisure choice, while equation (6) represents the equality between the rate of returns to human capital (the left-hand side) and the effective rate of interest (the interest rate on the debt plus the insurance premium the agent must pay when borrowing β).[10]

From this relationship, it follows that the time allocated to leisure is the same for all individuals regardless of their dates of birth: $l(s,t)=l(t)$. With all individual variables being additive across individuals, aggregate consumption equals

$$C(t)=\int_{-\infty}^t c(s,t)\beta e^{-\beta(t-s)}ds=(\varrho+\beta)\left[K(t)+\Omega(t)\right] \tag{7}$$

where $\Omega(t)\equiv\int_{-\infty}^t\omega(s,t)\beta e^{-\beta(t-s)}ds$ is the aggregate human wealth in the economy. The aggregate stock of physical capital is defined by

$$K(t)=\int_{-\infty}^t a(s,t)\beta e^{-\beta(t-s)}ds$$

[10]See Appendix A for further explanations.

and aggregate human capital is

$$H(t) = \int_{-\infty}^{t} h(s,t)\beta e^{-\beta(t-s)} ds, \tag{8}$$

2.2 Production sectors, the government and the environment

Two production sectors operate under perfect competition: one produces the final output denoted Q, while the other produces the abatement services denoted A. National income, measured in terms of final output, is

$$Y = Q + P_A A \tag{9}$$

where P_A is the relative price of abatement services in terms of final output production.

We assume that both the final output and the abatement services sectors are sources of pollution and that the government imposes a tax $\tau \in]0,1[$ on national income. Further, we consider that the tax revenue generated τY is completely used by the government to fund purchases of abatement services. Therefore

$$\tau Y = P_A A \tag{10}$$

From equation (9),

$$P_A A = \frac{\tau}{1-\tau} Q \qquad \text{and} \qquad Q = (1-\tau)Y \tag{11}$$

Final output Q is produced with the following technology:

$$Q = (\phi K)^{\alpha} (\psi H_p)^{1-\alpha}, \qquad \text{with} \;\; \phi, \;\psi, \;\alpha \;\in]0,1[$$

where $H_p \equiv \left[\int_{-\infty}^{t} u(s,t)h(s,t)\beta e^{-\beta(t-s)} ds\right]$ is the aggregate stock of human capital devoted to the production sectors. ϕ is the proportion of physical capital stock used in output production, and ψ is the proportion of H_p used in output production. Both human and physical capital are homogeneous and perfectly mobile across sectors. Firms in the final output sector maximise their profits $(1-\tau)Q - r\phi K - w\psi H_p$ by equating factor rewards to marginal productivity:

$$\begin{aligned} r &= \alpha(1-\tau)\frac{Q}{\phi K} \\ w &= (1-\alpha)(1-\tau)\frac{Q}{\psi H_p} \end{aligned} \tag{12}$$

The abatement sector produces abatement services aimed at curbing pollution emissions. Physical and human capital are then used in the abatement sector with the following constant returns technology:

$$A = [(1-\phi)K]^{\varepsilon}\,[(1-\psi)H_p]^{1-\varepsilon}\,, \qquad \text{with } \ \varepsilon \in]0,1[$$

Note that when $\varepsilon = \alpha$ the abatement services sector uses the same technology as the output sector, in other words, abatement services are produced with output. Profit maximisation in the abatement services sector yields

$$\begin{aligned} w &= (1-\varepsilon)(1-\tau)\frac{P_A A}{(1-\psi)H_p} \\ r &= \varepsilon(1-\tau)\frac{P_A A}{(1-\phi)K} \end{aligned} \tag{13}$$

From equations (13) and (11), we obtain:

$$r = \frac{\varepsilon\ \tau Q}{(1-\phi)K} \qquad \text{and} \qquad w = \frac{(1-\varepsilon)\ \tau Q}{(1-\psi)H_p}$$

From (12) we have $\frac{\alpha(1-\tau)Q}{\phi K} = \frac{\varepsilon\ \tau Q}{(1-\phi)K}$, that is

$$\phi = \frac{\alpha(1-\tau)}{\alpha + (\varepsilon - \alpha)\tau} \tag{14}$$

and $\frac{(1-\alpha)(1-\tau)Q}{\psi H_p} = \frac{(1-\varepsilon)\ \tau Q}{(1-\psi)H_p}$ gives

$$\psi = \frac{(1-\alpha)(1-\tau)}{(1-\alpha) - (\varepsilon - \alpha)\tau} \tag{15}$$

When $\varepsilon = \alpha$, we obtain $\phi = \psi = 1-\tau$ and therefore $Q = (1-\tau)K^{\alpha}H_p^{1-\alpha}$, $P_A A = \tau K^{\alpha} H_p^{1-\alpha}$ and $Y = K^{\alpha}H_p^{1-\alpha}$ like in Hettich (1998).

2.3 Balanced growth path (BGP) equilibrium

National income is used for three purposes: to finance abatement purchases (equal to τY), to consume, or to invest in physical capital. Therefore, the market clearing condition is

$$(1-\tau)Y = C + \dot{K}. \tag{16}$$

Differentiating (8) with respect to time and using the fact that $u(s,t) = u$,[11] the aggregate accumulation of human capital is

$$\dot{H} = B\,[1 - u - l]\,H \tag{17}$$

Because there is no tax on physical capital income we have $r_n = r$. By differentiating (7) with respect to time, using the expression of dK/dt, $d\Omega/dt$ and equation (4) we obtain

$$\dot{C}/C = r - \varrho - \beta\,(\varrho + \beta)\,K/C \tag{18}$$

The final term on the right-hand side represents the distributional effect of generational turnover. This effect arises because older generations consume more than younger generations. Therefore, when they die and are replaced by newborns (who hold no financial assets), aggregate consumption growth is reduced compared with individual consumption growth. Further, the generational turnover effect increases with the probability of dying β. On one hand, agents die at a higher frequency (which increases generational turnover). On the other hand the propensity to consume out of wealth $\varrho + \beta$ rises due to the shorter time horizon.

By defining $x \equiv C/K$, $z \equiv H/K$, the model can be summarised by the following three dynamical equations (see Appendix A):

$$\dot{x}/x = [\alpha(1-\tau) - \Phi(\tau)]\,\Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha} - \varrho - \beta(\beta+\varrho)x^{-1} + x \tag{19}$$

$$\dot{z}/z = B\,[1 - u - l] - \Phi(\tau)\Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha} + x \tag{20}$$

$$\dot{u}/u = \alpha^{-1}\left[B(1-l) - \beta - \alpha(1-\tau)\Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha}\right] - \dot{z}/z \tag{21}$$

with (from equations 14 and 15)

$$\Psi(\tau) \equiv \frac{\phi}{\psi} = \frac{\alpha}{1-\alpha}\left(\frac{1-\alpha-(\varepsilon-\alpha)\tau}{\alpha+(\varepsilon-\alpha)\tau}\right) \qquad \text{and} \qquad \Phi(\tau) \equiv \phi = \frac{\alpha(1-\tau)}{\alpha+(\varepsilon-\alpha)\tau}, \tag{22}$$

and by one static relationship (from equation 5):

$$l = \frac{\xi_l}{(1-\tau)(1-\alpha)} \times \frac{x\ u}{\Psi(\tau)^{\alpha}(z\ u)^{1-\alpha}} \tag{23}$$

The BGP equilibrium is a stationary equilibrium where $u = u^{\star}$, $z = z^{\star}$ and, $x = x^{\star}$ are defined by $\dot{x} = \dot{z} = \dot{u} = 0$ and $l = l^{\star}$.

[11] From (12), the equalisation of the rates of returns given by equation (6) implies that the rate of return to human capital is independent of s and therefore that all individuals allocate the same effort to schooling: $u(s,t) = u$.

Proposition 1. *Under the condition $B > \beta + \varrho$, for a particular value of τ, there is a unique BGP equilibrium along which $u^\star \in](\beta+\varrho)/B, 1[$ solves*

$$Bu^\star - \beta + \frac{1-\alpha}{\alpha}\Psi(\tau)\left(B\left(1-l^\star\right)-\beta\right) - \frac{\beta(\beta+\varrho)}{Bu^\star-\beta-\varrho} = 0$$

with $l^\star = \frac{1}{2B}\left(B-\beta-\sqrt{(B-\beta)^2 - 4\xi_l \frac{\alpha B\ \beta(\beta+\varrho)\ u^\star}{(1-\alpha)\Psi(\tau)(Bu^\star-\beta-\varrho)}}\right)$.

Proof. See Appendix B.1. ■

Further, along the BGP equilibrium the consumption to physical capital ratio is

$$x^\star = \frac{\beta(\beta+\varrho)}{Bu^\star-\beta-\varrho} > 0$$

and the human to physical capital ratio is

$$z^\star = \frac{\Psi(\tau)}{u^\star}\left[\frac{B(1-l^\star)-\beta}{\alpha(1-\tau)}\right]^{1/(1-\alpha)} > 0$$

Finally, the growth rate is

$$g^\star = B(1-u^\star-l^\star) > 0.$$

3 Environmental tax and growth rate

In this section, we investigate the relationship between environmental taxation and the rate of growth when the former is imposed on national income. We first examine the BGP equilibrium and then we study the transition of the economy after an increase in the tax rate in order to highlight the economic mechanisms that underlie how tax influences the BGP equilibrium.

3.1 Long term effects

Proposition 2. *In the presence of an environmental taxation on national income.*

a) When agents have finite life expectancy, tighter environmental tax promotes (respectively harms) human capital accumulation in the long term when the abatement sector is relatively more (resp. less) intensive in human capital compared with the final output sector. When technologies are the same across sectors, however, tighter environmental tax does not affect long term growth.

b) When agents have infinite life expectancy *(i) if labour supply is exogenous, environmental tax does not affect long term human capital accumulation.*

(ii) if labour supply is endogenous, tighter environmental tax promotes (respectively harms) human capital accumulation in the long term when the abatement sector is relatively more (resp. less) intensive in human capital compared with the final output sector.

Proof. See Appendix B.1 for 2a and Appendix B.2 for 2b. ■

Proposition 2 states that assuming a different technology for final output production and abatement services production leads to two important insights. First, environmental tax influences BGP growth. This result challenges conventional result that this tax does not affect the growth rate when pollution arises from the final output, in Lucas (1988) settings, as originally demonstrated by Gradus and Smulders (1993) and extended to the case of endogenous labour supply by Hettich (1998). Second, according to the relative factorial intensity of each sector, the influence of environmental tax may be positive or negative. When the abatement sector is more (*respectively less*) intensive in human capital than the final output sector, that is $\alpha > \varepsilon$ (*resp.* $\alpha < \varepsilon$), environmental tax enhances (*resp. reduces*) the BGP rate of growth. When the same technology is used for final production and abatement production ($\alpha = \varepsilon$), environmental tax does not affect BGP growth.

The intuition behind this result is as follows. At the impact, tighter environmental tax has two effects: *(i)* it reduces the rewards to physical capital (r) and to human capital (w) and *(ii)* it leads to a reallocation of factors between the final output sector and the abatement sector when the technologies used in the two sectors differ. Because tighter environmental tax leads to greater abatement activity, it creates a relative scarcity in the factors more intensively used in this sector and therefore its reward increases. Therefore, when the abatement sector is more (*resp. less*) intensive in human capital than the final output sector, more abatement activities require more human capital than is released by the output sector. The reward to human capital thus rises and agents are incited to invest in education.

When agents have an infinite life expectancy, the influence of environmental tax on the BGP rate of growth also depends on the existence of endogenous labour. Indeed, with an exogenous labour supply, human capital accumulation is given by $B(1 - u^\star)$ and must be equal to the growth rate of aggregate consumption $r^\star - \varrho$. Further, the returns to physical capital must be equal to the returns to human capital: $r^\star = B$.

Therefore, investment in education $1 - u^{\star}$ is independent of the environmental tax rate. Introducing an endogenous labour supply increases the incentive to invest in human capital if the human capital intensive sector is expanded by tighter environmental tax. This mechanism explains why the BGP rate of growth is boosted by environmental tax when the abatement sector is relatively more intensive in human capital.

When agents have a finite life expectancy, because the growth rate of aggregate consumption also relies on the generational turnover effect (see equation 18), even if labour supply is exogenous, environmental tax influences BGP human capital accumulation if the adopted technologies are different. For example, let us consider that the abatement sector is intensive in human capital and that environmental tax rises. Ceteris paribus, the growth rate of physical capital becomes smaller than those of human capital and consumption because of the generational turnover effect. Therefore, the ratio C/K rises and the growth rates of consumption and human capital increase (see equations 19 and 20).[12]

Nonetheless, Proposition 2a states that with finitely-lived agents environmental tax does not affect education in the long term when production technologies are the same. This finding contrasts with the result presented by Pautrel (2012a) when the source of pollution is physical capital. Our result suggests that the generational turnover effect is not sufficient by itself to generate a positive impact of the environmental tax on growth, because this tax is imposed on both physical and human capital (see Hettich, 1998). Therefore, physical capital accumulation and consumption growth simultaneously reduce, while the ratio C/K remains constant and the generational turnover effect stays the same. Therefore, agents do not modify their allocations of time to education and leisure, and because the interest rate is given by the equalisation of returns ($r^{\star} + \beta = B(1 - l^{\star})$ from equation 6), it remains constant. Finally, the after-tax rate of growth is the same as it was before the tightening of the environmental tax.

3.2 Short term effects

We next examine the influence of tighter environmental tax during the transition of the economy towards the new BGP equilibrium. Because the analytic study of the transition is cumbersome, we perform a numerical analysis using the time-elimination method (see Mulligan and Sala-i Martin, 1991, 1993). We calibrate the model to yield realistic values of the GDP growth rate for the US economy. According to the World Bank database

[12] I thank the co-editor Michael Rauscher who highlighted this point.

World Development Indicators 2005, life expectancy in the US was 78 years in 2005, while from the database of the Penn World Table 7.0 (Heston et al., 2011) real GDP growth rate was 3.5% during the period 1960-2005. Because expected life expectancy is the inverse of the probability of death per unit time β, we wish β to be close to $1/78 = 0.0128$. The weight of leisure in utility ξ_l is chosen to be 1.24, from Devereux and Love (1994). We adjust the other variables to obtain a benchmark growth rate of approximately 3.5% and $u^\star$ approximately 25% (to replicate Prescott, 2004, who documented the amount of time spent in hours worked per person aged 15-64 at the end of the 1990's to be close to 25%).[13]

We investigate the influence of technology during the transition by considering that, at time 0, the economy along the BGP experiences an unanticipated increase in environmental tax (from $\tau = 0.01$ to $\tau = 0.03$), for $\alpha > \varepsilon = 1/6$, $\alpha = \varepsilon = 1/3$, and $\alpha < \varepsilon = 1/2$ respectively.[14] In the first case (*resp. the last case*) the abatement sector is relatively more (*resp. less*) intensive in human capital. Table 1 presents the benchmark parameter values, while Table 2 summarises the comparative statics.

Table 1: Benchmark parameter values

α	ε	ϱ	B	β	ξ_l
1/3	1/3	0.025	0.14	0.0128	1.24

Table 2: Increase in environmental tax along the BGP

ε	1/6		1/3		1/2	
τ	0.01	0.03	0.01	0.03	0.01	0.03
$g^\star(\%)$	3.793	3.827	3.775	3.775	3.757	3.721
$u^\star$	0.292	0.291	0.292	0.292	0.292	0.293
$l^\star$	0.438	0.436	0.439	0.439	0.440	0.442
$x^\star$	0.161	0.163	0.160	0.161	0.158	0.156
$z^\star$	0.309	0.326	0.305	0.314	0.302	0.304
$r^\star + \beta$	0.0787	0.0790	0.0786	0.0786	0.0784	0.0781

[13]The BGP equilibrium we computed has two positive eigenvalues and one negative eigenvalue. Because there are two jump variables (x and u) and one predetermined variable (z), the equilibrium is locally saddle-point stable. Because of the 3×3 matrix, the analytical demonstration of the saddle-path stability is cumbersome and is beyond the scope of this paper.

[14]The results are robust to alternative values of ε.

Graph 1 (at the end of the paper) draws the evolution of the main variables towards the new steady-state due to the unanticipated increase in environmental tax. We report the temporal evolution of the variables deflated by their respective before-tax steady-state level.[15]

When the abatement sector is relatively intensive in human capital, an increase in environmental tax leads to a rise in abatement services production, which requires more human capital. Because human capital is freed from the output sector which is relatively more intensive in physical capital, there is higher pressure on human capital rewards: w does not fall instantaneously as it did with the same factor intensity in both sectors (Graph 1.ii). Conversely r declines to a larger degree because more physical capital is released relatively for every input of human capital reallocated from the output sector to the abatement sector (Graph 1.i). As a consequence, the fall in the aggregate consumption to physical capital ratio C/K is instantaneously lowered (Graph 1.iii), while the decline in u is higher because the gap between returns is greater (Graph 1.v). Therefore the jump of l is reduced (Graph 1.vi). Adjustment mechanisms towards the new BGP equilibrium are similar to the case of $\alpha = \varepsilon$: C/K and H/K rise, whereas l falls. Nevertheless these variations are magnified and C/K and l shift respectively above and below their initial values (Graph 1.iii & vi). Human capital accumulation is thus boosted, while the accumulation of physical capital drops further to the point that the equalisation of returns is made for a value of u lower than the initial value as shown in Graph 1.v (this also occurs because of a greater increase in H/K).

Further, there is a trade–off between the short and long term when we consider how environmental tax influences growth. Growth decreases in the short term but increases in the long term when the abatement sector is more intensive in human capital than the final output sector (Graph 1.viii) (this is also true for C/K, u, and l). Such a trade–off is explained by the fact that, at the point when tax is increased, the interest rate instantaneously drops below its initial value but remains above its initial value along the new BGP equilibrium.

By contrast, when the abatement sector is relatively intensive in physical capital ($\alpha < \varepsilon$), the drop in w is higher while the decrease in the interest rate is limited (Graph 1.i & ii). Relatively speaking, more human capital is released for every input of physical capital reallocated from the output sector to the abatement sector. Therefore, the instantaneous

[15] That enables us to draw on the same graph the different cases: abatement sector more intensive in human capital ($\varepsilon < \alpha$), abatement sector more intensive in physical capital ($\varepsilon > \alpha$), same technology ($\varepsilon = \alpha$).

fall in C/K is higher (Graph 1.iii) and that in u is smaller (Graph 1.v). Because the abatement sector is relatively more intensive in physical capital, the substitution between these two types of capital is reduced (compared with the case when the technologies are similar) and H/K rises to a lower degree (Graph 1.iv). Further, u rises above its initial value. Finally the equalisation of returns is made for a value of the interest rate lower than its initial value (after the tightening of environmental tax). The growth rate along the new BGP equilibrium is thus lower than its initial value (Graph 1.viii) and tighter environmental tax reduces growth in the long term. The presented numerical simulations also show that there is no trade–off between the short and long term in terms of how environmental tax influences growth when the abatement sector is more intensive in physical capital, because for each unit of physical capital required in the abatement sector, a tiny amount of physical capital is freed from the production sector.[16]

4 Environmental tax applied to physical capital income

In the previous section we demonstrated that environmental policy affects long-term growth in a Lucas (1988)' model when environmental tax is applied to national income if the production technology in the abatement sector differs from that in the final output sector. We also showed that this influence of environmental policy on growth could be either positive or negative. The purpose of this section is to investigate whether the results found previously remain valid when environmental tax is applied to physical capital income.

We assume that physical capital income is taxed at a rate τ_k. Therefore, the after-tax real interest rate is $r_n = (1-\tau_k)\, r$ and equation (10) becomes

$$\tau_k r K = P_A A \tag{24}$$

Furthermore, equations (12) and (13) become

$$r = \alpha \frac{Q}{\phi K} = \varepsilon \frac{P_A A}{(1-\phi)K} \tag{25a}$$

$$w = (1-\alpha)\frac{Q}{\psi H_p} = (1-\varepsilon)\frac{P_A A}{(1-\psi)H_p} \tag{25b}$$

[16]Note that we find a trade–off between the short and long terms when the environmental tax stimulus is less important and the difference in factor intensity is high enough ($\varepsilon >> \alpha$). Indeed, in this case an increase in abatement production due to a tighter environmental tax reinforces the positive pressure on the interest rate.

Using (24) and dividing member by member equations (25a) and (25b), we obtain

$$\frac{1-\psi}{\psi} = \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{\alpha}{1-\alpha}\right)\frac{1-\phi}{\phi} \qquad \text{that is} \qquad \psi = \frac{(1-\alpha)\varepsilon}{(\varepsilon-\alpha)\phi+\alpha(1-\varepsilon)}\phi$$

From (24), we also have $r = \varepsilon\frac{\tau_k r K}{(1-\phi)K}$, that is

$$\phi = 1-\varepsilon\tau_k > 0$$

When abatement services are produced with the same technology as for the final output ($\varepsilon = \alpha$), we obtain $\phi = \psi = 1-\alpha\tau_k$. Therefore, $Q = (1-\alpha\tau_k)K^\alpha H_p^{1-\alpha}$, $P_D D = \alpha\tau_k\ K^\alpha H_p^{1-\alpha}$ and $Y = K^\alpha H_p^{1-\alpha}$.

Finally, from (25a), $r\tau_k K = \left(\frac{\alpha}{\varepsilon}\right)\left(\frac{1}{\phi}-1\right)Q$. Because final output is used for consumption and physical capital investment, the market clearing condition yields

$$\left(1+\left(\frac{\alpha}{\varepsilon}\right)\left(\frac{1}{\phi}-1\right)\right)Q = C+\dot{K}$$

Therefore, the dynamic system is[17]

$$\dot{x}/x = \left[\alpha(1-\tau_k)-1-(\alpha-\varepsilon)\tau_k\right]\Upsilon(\tau_k)^{\alpha-1}(z\ u)^{1-\alpha}-\varrho-\beta(\beta+\varrho)x^{-1}+x \tag{26}$$

$$\dot{z}/z = B(1-u-l)-(1+(\alpha-\varepsilon)\tau_k)\Upsilon(\tau_k)^{\alpha-1}(z\ u)^{1-\alpha}+x \tag{27}$$

$$\dot{u}/u = \alpha^{-1}\left[B(1-l)-\beta-\alpha(1-\tau_k)(z\ u)^{1-\alpha}\right]-\dot{z}/z \tag{28}$$

where $\Upsilon(\tau_k) \equiv \frac{\phi}{\psi} = 1+\frac{\alpha-\varepsilon}{1-\alpha}\tau_k$ (with $\Upsilon_{\tau_k}(\tau_k) \geq 0\ \ if\ \ \alpha \geq \varepsilon$), and

$$l = \frac{\xi_l}{(1-\alpha)\Upsilon(\tau_k)^\alpha}\times\frac{x\ u}{(z\ u)^{1-\alpha}} \tag{29}$$

Solving the dynamic system provides us with the following proposition:

Proposition 3. *If environmental tax is applied to physical capital income, tighter environmental tax promotes long-term human capital accumulation regardless of the relative factor intensity in the abatement and final output sectors.*

Proof. See Appendix B. ∎

Proposition 3 states that abatement technology does not influence the growth effect of environmental policy when environmental tax is applied to physical capital income. This result can be explained as follows. Let us suppose that the abatement sector is relatively more intensive in physical capital. When physical capital income is taxed, the relative

[17] These equations are derived in a similar way to equations (19)–(21).

factor reward r/w drops to a greater degree than when output is taxed. Therefore, the time allocated to production (u) diminishes further and the fall in the interest rate is so high that even if physical capital were relatively scarce in output production (because abatement production is relatively more intensive in physical capital), the rising force due to that scarcity would not be sufficient to drive the interest rate upwards at the impact.[18] Therefore, the overall adjustment mechanisms remain the same regardless of the relative factorial intensity in the final output and abatement sectors.

5 Concluding remarks

The present paper re-examined the growth effects of tighter environmental tax when imposed on national income and when human capital accumulation is the engine of growth. Compared with previous contributions, we have accounted for finitely-lived agents and relaxed the basic assumption that production technologies are similar in the final output and abatement sectors.

The main insight of this paper is that the choice of abatement technology is important when the environmental tax is applied to national income. Thanks to reallocation effects, environmental tax affects short and long term growth according to the relative factor intensity between the final goods sector and the abatement sector: growth increases in the long term and decreases in the short term if the abatement sector is more intensive in human capital than the final output sector. Because it seems to be the relevant case for developed countries, the present paper suggests that short term costs should be carefully accounted before the implementation of the environmental policy. Furthermore, this paper demonstrates that the role played by the generational turnover effect (arising from the overlapping generations à la Yaari (1965)–Blanchard (1985)) is insufficient by itself to allow environmental tax to influence growth positively when the final output is taxed.

By highlighting how the structure of the economy at the sectorial level influences the economic outcome of the implementation of the environmental tax policy, this paper calls for an empirical investigation into factorial intensity in polluting sectors and into

[18] Even in the most disadvantageous case, namely when the abatement sector uses no human capital at all, the initial drop in the interest rate would be smaller but would remain negative (see equation 25a for the limit case $\varepsilon = 0$). There is no reallocation of human capital from the output sector to the abatement sector however because the returns to physical capital investment are smaller than those to education at the impact, agents are incited to invest more in human capital accumulation. The process is made easier because physical capital moves from the output sector to the abatement sector and because the abatement sector does not use human capital therefore, human capital is freed from the output sector to be employed in education.

abatement activities in a range of countries. Indeed, it is well documented that in both developed and developing countries, the highest polluting industries are those that have the higher physical capital to human capital ratios.[19] Because there are several types of pollution (e.g., air, water, etc) and because abatement technologies for each type aim to reduce or prevent their damaging impact, defining capital intensity in the abatement sector can be complicated. OECD/Eurostat (1999, Appendix 2, pp.39–44) offers a non-exhaustive list of the environmental goods that contribute to curbing pollution. Some such goods are clearly relatively more intensive in physical capital (with respect to human capital) while others are the opposite. Empirical data are therefore needed before political recommendations for specific countries can be made.

This paper also contributes to the body of knowledge on this topic by specifying and enriching the findings of previous seminal works, and therefore, calls for more interest in modeling the abatement "*side*" of the growth-environment relationship and in the role played by finite lifetimes. Nevertheless, it suffers from some limitations. Our results are limited to the case of Cobb–Douglas production functions and logarithmic utility function. It would be interesting to investigate wether these change when general specifications are used. Introducing a more realistic demographic structure (e.g., by relaxing the perpetual youth assumption of the Yaari (1965)–Blanchard (1985) model as in Heijdra and Mierau (2010)) could also improve our understanding of the role played by finite lifetimes in the environment-growth nexus. Moreover, the assumption of an imperfect market for actuary notes could be introduced into our model in order to examine how the quantitative impact of environmental tax on growth alters. Finally, the provision of abatement goods and services from abroad should be introduced by enlarging our framework to open economies in order to take into account the rise in the trade of environmental goods (Kennett and Steenblik, 2005) .

[19] According to Mani et al. (2002), five sectors emerge as dirty industries: Iron and Steel, Non-Ferrous Metals, Industrial Chemicals, Paper and Pulp (341) and Non-Metallic Mineral Products.The authors exclude Petroleum because very few countries are producers. The majority of environmental works that have tested the "pollution haven" hypothesis have calculated the factorial intensity of the highest polluting industries for the US (Cole et al., 2005), Japan (Cole et al., 2006) and China (Cole et al., 2008). These studies have found that the physical to human capital ratio is higher than unity for these nations.

References

Barro, R. and Sala-i Martin, X. (1995). *Economic Growth*. MacGraw-Hill, New-York.

Blanchard, O. and Fisher, S. (1989). *Lectures on Macroeconomics*. MIT Press, cambridge, MA.

Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy*, 93(2):223–247.

Bovenberg, A. and Van Ewijk, C. (1997). Progressive taxes, equity, and human capital accumulation in an endogenous growth model with overlapping generations. *Journal of Public Economics*, 64:153–179.

Cole, M. A., Elliott, R. J. R., and Shimamoto, K. (2005). Why the grass is not always greener: The competing effects of environmental regulations and factor intensities on us specialization. *Ecological Economics*, 54(1):95–109.

Cole, M. A., Elliott, R. J. R., and Shimamoto, K. (2006). Globalization, firm-level characteristics and environmental management: A study of japan. *Ecological Economics*, 59(3):312–323.

Cole, M. A., Elliott, R. J. R., and Wu, S. (2008). Industrial activity and the environment in china: An industry-level analysis. *China Economic Review*, 19(3):393–408.

Devereux, M. B. and Love, D. R. F. (1994). The effects of factor taxation in a two-sector model of endogenous growth. *Canadian Journal of Economics*, 27(3):509–536.

Gradus, R. and Smulders, S. (1993). The trade-off between environmental care and long-term growth-pollution in three prototype growth models. *Journal of Economics*, 58(1):25–51.

Grimaud, A. and Tournemaine, F. (2007). Why can environmental policy tax promote growth through the channel of education? *Ecological Economics*, 62(1):27–36.

Heijdra, B. (2009). *Foundations of Modern Macroeconomics*. Oxford University Press.

Heijdra, B. J. and Mierau, J. (2010). Growth effects of consumption and labor-income taxation in an overlapping-generations life-cycle model. *Macroeconomic Dynamics*, 14(Supplement S2):151–175.

Heston, A., Summers, R., and B. Aten, P. (2011). Penn world table version 7.0. Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania.

Hettich, F. (1998). Growth effects of a revenue-neutral environmental tax reform. *Journal of Economics*, 67(3):287–316.

Kennett, M. and Steenblik, R. (2005). Environmental goods and services: a synthesis of country studies. OECD Trade and Environment Working Papers 2005/03, OECD.

Lucas, Robert E., J. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42.

Mani, M., Wheeler, D., and Munasinghe, M. (2002). *In Search of Pollution Havens? Dirty Industry in the World Economy, 1960-1995*, pages 464–477. Elgar Reference Collection. International Library of Critical Writings in Economics, vol. 141., Unlisted.

Miller, M. H. and Modigliani, F. (1961). Dividend policy, growth, and the valuation of shares. *The Journal of Business*, 34(4):411–433.

Mulligan, C. and Sala-i Martin, X. (1991). A note on the time-elimination methods for solving recursive dynamic economic models. National Bureau of Economic Research Technical Working Paper 116, Cambridge MA.

Mulligan, C. B. and Sala-i Martin, X. (1993). Transitional dynamics in two-sector models of endogenous growth. *Quarterly Journal of Economics*, 108(3):739–773.

Nimubona, A.-D. and Sinclair-Desgagné, B. (2011). Polluters and abaters. *Annals of Economics and Statistics / Annales d'Economie et de Statistique*, (103/104):9–24.

OECD/Eurostat (1999). *The Environmental Goods and Services Industry: manual for data collection and analysis.* OECD, Paris.

Pautrel, X. (2008). Reconsidering the impact of pollution on long-run growth when pollution influences health and agents have a finite-lifetime. *Environmental and Resource Economics*, 40(1):37–52.

Pautrel, X. (2009). Pollution and life expectancy: How environmental policy can promote growth. *Ecological Economics*, 68(4):1040–1051.

Pautrel, X. (2012a). Environmental policy, education and growth: A reappraisal when lifetime is finite. *Macroeconomic Dynamics*, 16(5):661–685.

Pautrel, X. (2012b). Pollution, private investment in healthcare, and environmental policy. *The Scandinavian Journal of Economics*, 114(2):334–357.

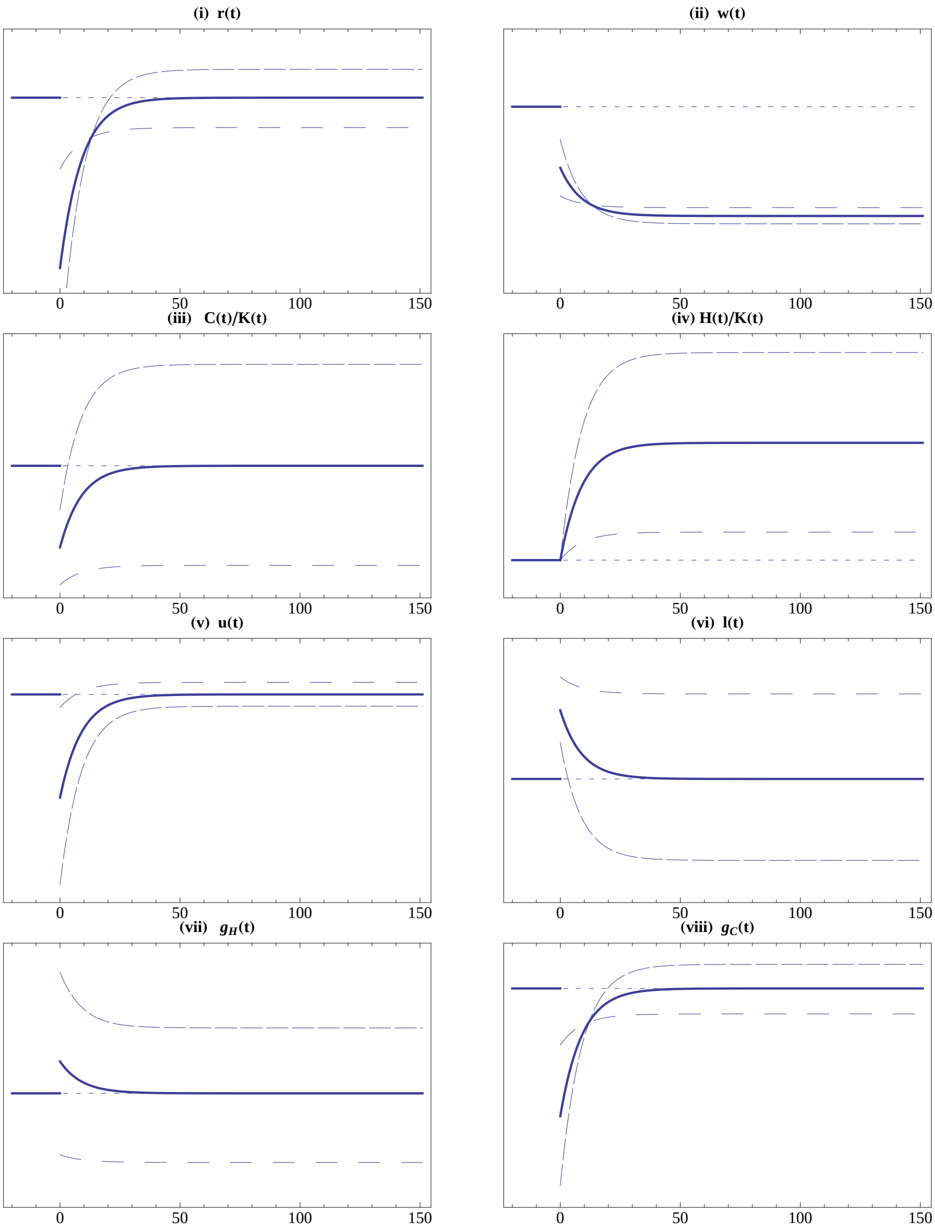
Prescott, E. C. (2004). Why do americans work so much more than europeans? *Federal Reserve Bank of Minneapolis Quarterly Review*, 28(1):2–13.

Sinclair-Desgagne, B. (2008). The environmental goods and services industry. *International Review of Environmental and Resource Economics*, 2(1):69–99.

Van Ewijk, C. and Van Wijnbergen, S. (1995). Can abatement overcome the conflict between environment and economic growth? *De Economist*, 143(2):197–216.

Yaari, M. (1965). Uncertain lifetime, life insurance and the theory of consumer. *Review of Economic Studies*, 32:137–150.

Graph 1. Short and long term influence of tighter environmental tax



−− $\varepsilon < \alpha$: Abatement sector is more intensive in human capital,

− − $\varepsilon > \alpha$: Abatement sector is more intensive in physical capital,

− $\varepsilon = \alpha$: Same technology.

Variables are deflated by their respective initial (before-tax) level

Appendix A

In this appendix, we solve the model.

The program of the households is

$$\begin{aligned}
\max_{c(s,t),l(s,t),a(s,t),h(s,t),u(s,t)} \quad & \int_s^\infty \left[\log c(s,t) + \xi_l \log l(s,t) - \zeta \log S\right] e^{-(\varrho+\beta)(t-s)} dt \\
s.t. \quad & \dot{a}(s,t) = \left[r_n + \beta\right] a(s,t) + u(s,t)h(s,t)w(t) - c(s,t) \\
& \dot{h}(s,t) = B\left[1 - u(s,t) - l(s,t)\right] h(s,t) \\
& a(s,s) = 0 \qquad h(s,s) = H(s) > 0
\end{aligned}$$

The Hamiltonian of the program may be written as

$$\begin{aligned}
\mathcal{H} = \log c(s,t) + \xi_l \log l(s,t) - \zeta \log S + \pi_1(t)\left[(r_n + \beta)\, a(s,t) + u(s,t)h(s,t)w(t) - c(s,t)\right] \\
+ \pi_2(t) B\left[1 - u(s,t) - l(s,t)\right] h(s,t)
\end{aligned}$$

where $\pi_1(t)$ and $\pi_2(t)$ are the co-state variables.

The first-order conditions are

$$\frac{\partial \mathcal{H}}{\partial c(s,t)} = 0 \qquad \Rightarrow \qquad \frac{1}{c(s,t)} = \pi_1(t) \tag{30}$$

$$\frac{\partial \mathcal{H}}{\partial u(s,t)} = 0 \qquad \Rightarrow \qquad \pi_1(t)w(t) = \pi_2(t)B \tag{31}$$

$$\frac{\partial \mathcal{H}}{\partial l(s,t)} = 0 \qquad \Rightarrow \qquad \frac{\xi_l}{l(s,t)} = \pi_2(t)Bh(s,t) \tag{32}$$

$$\frac{\partial \mathcal{H}}{\partial a(s,t)} = -\dot{\pi}_1(t) + (\varrho+\beta)\pi_1(t) \qquad \Rightarrow \qquad \pi_1(t)(r(t)+\beta) = -\dot{\pi}_1(t) + (\varrho+\beta)\pi_1(t) \tag{33}$$

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial h(s,t)} = -\dot{\pi}_2(t) + (\varrho + \beta)\pi_2(t) \qquad \Rightarrow \\
\pi_1(t)w(t)u(s,t) + \pi_2(t)B(1 - u(s,t) - l(s,t)) = -\dot{\pi}_2(t) + (\varrho + \beta)\pi_2(t)
\end{aligned} \tag{34}$$

From (30) and (33), we obtain

$$\dot{c}(s,t) = (r(t) - \varrho)c(s,t) \tag{4}$$

Equations (30), (31) and (32) give

$$l(s,t) = \frac{\xi_l c(s,t)}{w(t)h(s,t)} \tag{35}$$

From equation (33): $\varrho + \beta = r(t) + \beta + \frac{\dot{\pi}_1(t)}{\pi_1(t)}$. From equations (31), (33) and (34): $\varrho + \beta = B(1 - l(s,t)) + \frac{\dot{\pi}_2(t)}{\pi_2(t)}$. Equalizing both expression we obtain

$$\frac{B(1 - l(s,t))\pi_2(t) + \dot{\pi}_2(t)}{\pi_2(t)} = \frac{(r(t) + \beta)\pi_1(t) + \dot{\pi}_1(t)}{\pi_1(t)} \tag{36}$$

First, this equation means that $l(s,t)$ is independent from s: $l(s,t) = l(t)$ defined by equation (35). Second, this equation is the "*fundamental principle of valuation*" (Miller and Modigliani, 1961, p. 412) according to which the rate of return on different assets (dividends plus capital gains) must be equal.[20] $B(1-l(t))\pi_2(t)$ is the dividend on human capital and $(r(t)+\beta)\pi_1(t)$ is the dividend on physical capital (or financial assets) valued in terms of utility. The dividend on human capital is explained as follows (see equation 34): Investing in one unit of human capital rises the stock of human capital by $B(1 - u - l)$ valued in terms of utility at π_2 the shadow price of human capital, and increases earnings from production by uw valued in terms of utility at π_1 the shadow price of physical capital (or financial assets). Therefore the dividend on human capital is $B(1 - u - l)\pi_2 + uw\pi_1$. Because time must be equally valuable in its two uses (physical capital accumulation and human capital accumulation), we have $\pi_1 w = \pi_2 B$ (see equation 31). Therefore, the dividend on human capital is equal to $B(1-l)\pi_2$. Differentiating (31) with respect to time, we obtain $\frac{\dot{\pi}_1(t)}{\pi_1(t)} + \frac{\dot{w}(t)}{w(t)} = \frac{\dot{\pi}_2(t)}{\pi_2(t)}$. Replacing by the expressions of $\frac{\dot{\pi}_1(t)}{\pi_1(t)}$ and $\frac{\dot{\pi}_2(t)}{\pi_2(t)}$, it gives

$$\frac{\dot{w}(t)}{w(t)} + B(1 - l(t)) = r(t) + \beta$$

which is an another way to write the fundamental principle of valuation. Because the return to physical capital is independent of s, the return to human capital is also independent of s and, therefore, agents allocate the same effort to schooling: $u(s,t) = u(t)$.

Differentiating the expression of aggregate consumption, $C(t) = \int_{-\infty}^{t} c(s,t)\beta e^{-\beta(t-s)} ds$, with respect to time, using the expression of dK/dt, $d\Omega/dt$ and equation (4) we obtain

$$\dot{C}/C = r - \varrho - \beta\left(\varrho + \beta\right) K/C \tag{37}$$

The market clearing condition is written as

$$\dot{K}(t) = (1 - \tau)Y(t) - C(t)$$

[20]This explanation is borrowed from Heijdra (2009, p.468).

Recalling that $\dot{h}(s,t) = B(1-u(t)-l(t))h(s,t)$ and $h(s,s) = H(s)$, differentiating with respect to time $H(t) = \int_{-\infty}^{t} h(s,t)\beta e^{-\beta(t-s)}ds$ gives the aggregate accumulation of human capital:

$$\dot{H}(t) = B(1-u(t)-l(t))H(t) \tag{17}$$

Finally, from equation (6), from the expression of the wage rate (equation 13) and the fact that $\int_{-\infty}^{t} u(s,t)h(s,t)\beta e^{-\beta(t-s)}ds = u(t)H(t)$ because $u(s,t) = u(t)$, we obtain

$$\frac{\dot{u}(t)}{u(t)} = \frac{\dot{K}(t)}{K(t)} - \frac{\dot{H}(t)}{H(t)} - \alpha^{-1}\left[r(t) + \beta - B(1-l(t))\right]$$

Defining $x \equiv C/K$ and $z \equiv H/K$, and using previous results, the dynamical system is summarized by

$$\dot{x}/x = \left[\alpha(1-\tau) - \Phi(\tau)\right]\Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha} - \varrho - \beta(\beta+\varrho)x^{-1} + x \tag{19}$$

$$\dot{z}/z = B\left[1-u-l\right] - \Phi(\tau)\Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha} + x \tag{20}$$

$$\dot{u}/u = \alpha^{-1}\left[B(1-l) - \beta - \alpha(1-\tau)\Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha}\right] - \dot{z}/z \tag{21}$$

Appendix B

B.1 The case $\beta > 0$

From (21), $\dot{u} = 0$, we obtain

$$(z^\star\ u^\star)^{1-\alpha} = \frac{\Psi(\tau)^{1-\alpha}}{\alpha(1-\tau)}\left[B(1-l^\star) - \beta\right] \tag{38}$$

Because $z^\star\ u^\star > 0$, it implies

$$B(1-l^\star) - \beta > 0 \qquad \Rightarrow \qquad B > \beta \tag{39}$$

The returns to education in the long term $B(1-l^\star)$ net of the probability to die β must be positive in order to incite agents to invest in human capital accumulation, and therefore to have a positive rate of growth along the balanced growth path.

Furthermore, from (19) and (20), $\dot{x} - \dot{z} = 0$ at the steady-state gives (with 38):

$$x^\star = \frac{\beta(\beta+\varrho)}{Bu^\star - \beta - \varrho} \tag{40}$$

Because $x^\star > 0$, it is required that

$$u^\star > \frac{\beta+\varrho}{B} \qquad \text{with} \quad B > \beta + \varrho \tag{41}$$

$B > \beta + \varrho$ means that the productivity in education B must be higher than the subjective discount rate $\varrho + \beta$ (the rate of time preference plus the instantaneous probability of death) in order to incite agents to give up one unit of consumption today and invest it in education. Conditions (39) and (41) are conventional in the Lucas (1988) human capital accumulation model.[21] Therefore, $x^\star\ u^\star$ is an increasing function of $u^\star$.

Using (23) and (38), we obtain $l^\star = \frac{\xi_l \alpha}{(1-\alpha)\Psi(\tau)} \times \frac{x^\star\ u^\star}{B(1-l^\star)-\beta}$ which is a quadratic equation of $l^\star \in [0,1[$ with only one solution verifying $B(1-u^\star-l^\star) > 0$:

$$l^\star = \frac{1}{2}\left(\frac{B-\beta-\sqrt{(B-\beta)^2 - 4\frac{\xi_l\alpha}{(1-\alpha)\Psi(\tau)}B\ x^\star\ u^\star}}{B}\right) \tag{42}$$

with $\partial l^\star/\partial u^\star > 0$ (because $x^\star u^\star$ is increasing in $u^\star$ as demonstrated above) and $\partial l^\star/\partial\Psi(\tau) < 0$.

$\dot{z} = 0$ leads to, using (20), (38) and (22):

$$x^\star = Bu^\star - \left(1 + \frac{1-\alpha}{\alpha}\Psi(\tau)\right)\beta + \frac{1-\alpha}{\alpha}\Psi(\tau)B\left[1-l^\star\right] \tag{43}$$

Equating (40) and (43) enables to express $u^\star$ as the solution of

$$\Gamma(u,\tau) \equiv Bu - \beta + \frac{1-\alpha}{\alpha}\Psi(\tau)\left(B\left(1-l^\star\right)-\beta\right) - \frac{\beta(\beta+\varrho)}{Bu-\beta-\varrho} = 0$$

with $l^\star$ defined by equation (42), and $\Gamma_u(u,\tau) > 0$ and $\Gamma_{\Psi(\tau)}(u,\tau) > 0$ because $\Gamma_u(u,\tau) = B + \frac{B\beta(\beta+\varrho)}{(Bu-\beta-\varrho)^2} + \xi_l \frac{\frac{B\beta(\beta+\varrho)^2}{(Bu-\beta-\varrho)^2}}{\sqrt{(B-\beta)^2 - \frac{4\alpha\xi_l}{(1-\alpha)\Psi(\tau)}B\ x^\star u^\star}} > 0$. From (41), $u^\star \in](\beta+\varrho)/B, 1[$ with $B > \beta+\varrho$. We have $\lim_{u\to(\beta+\varrho)/B} = -\infty$ and $\lim_{u\to 1} > 0$ because $B-\beta-\varrho > \beta(\beta+\varrho)$ (sufficient condition). Therefore, $u^\star \in](\beta+\varrho)/B, 1[$ is unique.

From the theorem of the implicit function, $u^\star$ is a decreasing function of $\Psi(\tau)$ and because from (22), we have $\Psi_\tau(\tau) \gtreqless 0$ when $\alpha \gtreqless \varepsilon$, therefore it comes

$$u^\star = \mathcal{U}(\tau) \qquad \text{with} \qquad \mathcal{U}_\tau(\tau) \leq 0 \ \text{ when } \ \alpha \geq \varepsilon$$

From (42), we obtain that

$$l^\star = \mathcal{L}(\tau) \qquad \text{with} \ \ \mathcal{L}_\tau(\tau) \leq 0 \ \text{ when } \ \alpha \geq \varepsilon$$

and

$$g^\star = B(1-\mathcal{U}(\tau)-\mathcal{L}(\tau)) \qquad \text{with} \ \ g^\star_\tau \geq 0 \ \text{ when } \ \alpha \geq \varepsilon$$

[21] See the textbook by Barro and Sala-i Martin (1995, p.184), among others.

B.2 The case $\beta = 0$

When life expectancy is infinite, $\beta = 0$ and the five equations (19-21, 22, 23) becomes:

$$\dot{x}/x = [\alpha(1-\tau) - \Phi(\tau)]\, \Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha} - \varrho + x \tag{45}$$

$$\dot{z}/z = B(1-u-l) - \Phi(\tau)\Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha} + x \tag{46}$$

$$\dot{u}/u = \alpha^{-1}\left[B(1-l) - (1-\tau)\alpha\Psi(\tau)^{\alpha-1}(z\ u)^{1-\alpha}\right] - \dot{z}/z \tag{47}$$

To obtain the expression of the BGP rate of growth, just remember that $g^\star = r^\star - \varrho$ where $r^\star$ is the interest rate along the BGP defined as $r^\star = (1-\tau)\alpha\Psi(\tau)^{\alpha-1}(z^\star\ u^\star)^{1-\alpha}$. Therefore, along the BGP $\dot{u} = 0$, implies that

$$r^\star = B(1-l^\star) \qquad \Rightarrow \qquad l^\star = 1 - \frac{r^\star}{B}$$

Furthermore, from (45),

$$x^\star = \varrho - \left[1 - \frac{\Phi(\tau)}{\alpha(1-\tau)}\right]\ r^\star = \varrho + \Psi(\tau)r^\star$$

and $\dot{x} - \dot{z} = 0$ leads to

$$u^\star = \frac{\varrho}{B}$$

Therefore equation (23) gives the implicit expression of $r^\star$:

$$1 - \frac{r^\star}{B} = \xi_l \frac{[\varrho + \Psi(\tau)r^\star]\ \varrho/B}{\left(\frac{1-\alpha}{\alpha}\right)\Psi(\tau)\ r^\star}$$

that is

$$B - r^\star = \frac{\xi_l}{\left(\frac{1-\alpha}{\alpha}\right)\ \Psi(\tau)}\left[\frac{\varrho}{r^\star} + \Psi(\tau)\right]\ \varrho$$

whose the unique solution such that $r^\star - \varrho > 0$ is:

$$r^\star = \frac{(1-\alpha)B - \xi_l\alpha\varrho + \sqrt{((1-\alpha)B - \xi_l\alpha\varrho)^2 - 4\xi_l\alpha(1-\alpha)\varrho^2/\Psi(\tau)}}{2(1-\alpha)} > 0$$

It is straightforward that $\partial r^\star/\partial\Psi(\tau) > 0$, $\forall \xi_l > 0$. Because $\partial\Psi(\tau)/\partial\tau \geq 0$ if and only if $\alpha \geq \varepsilon$ then $\forall\xi_l > 0$, $\partial r^\star/\partial\tau \geq 0$ if and only if $\alpha \geq \varepsilon$. When $\xi_l = 0$, $r^\star = B$ independent from τ.

Appendix C: Environmental tax is applied to physical capital income

From (28), $\dot{u} = 0$, we obtain

$$(z^\star\ u^\star)^{1-\alpha} = \frac{\Upsilon(\tau_k)^{1-\alpha}}{\alpha(1-\tau_k)}\left[B(1-l^\star) - \beta\right] \tag{48}$$

with $\Upsilon(\tau_k) \equiv 1 + \frac{\alpha-\varepsilon}{1-\alpha}\tau_k$. Because $z^\star\ u^\star > 0$, condition (39) $B > \beta$ always holds.

From (26) and (27), $\dot{x} - \dot{z} = 0$ at the steady-state gives (with 48):

$$x^\star = \frac{\beta(\beta+\varrho)}{Bu^\star - \beta - \varrho} \tag{49}$$

and because condition (41) always holds, $x^\star\ u^\star$ is an increasing function of $u^\star$

Using (23) and (48), we obtain $l^\star = \frac{\xi_l \alpha(1-\tau_k)}{(1-\alpha)\Upsilon(\tau_k)} \times \frac{x^\star\ u^\star}{B(1-l^\star)-\beta}$ which is a quadratic equation of $l^\star \in]0,1[$ with only one solution verifying $B(1-u^\star-l^\star) > 0$:[22]

$$l^\star = \frac{1}{2}\left(\frac{B - \beta - \sqrt{(B-\beta)^2 - 4\frac{\alpha\xi_l(1-\tau_k)}{(1-\alpha)\Upsilon(\tau_k)}B\ x^\star\ u^\star}}{B}\right) \tag{50}$$

with $\partial l^\star/\partial u^\star > 0$ and $\partial l^\star/\partial \tau_k \leq 0$.

$\dot{z} = 0$ leads to, using (27) and (48):

$$x^\star = Bu^\star - \beta + \left(\frac{1+(\alpha-\varepsilon)\tau_k}{\alpha(1-\tau_k)} - 1\right)\left[B\left(1-l^\star\right) - \beta\right] \tag{51}$$

Equating (49) and (51) enables to express $u^\star$ as the solution of

$$Bu^\star - \beta + \left(\frac{1+(\alpha-\varepsilon)\tau_k}{\alpha(1-\tau_k)} - 1\right)\left[B\left(1-l^\star\right) - \beta\right] - \frac{\beta(\beta+\varrho)}{Bu - \beta - \varrho} = 0$$

with $l^\star$ defined by equation (50). The LHS is increasing in $u^\star$ and τ_k $\forall$ $(\alpha,\varepsilon) \in]0,1[$, because $\partial\left(\frac{1+(\alpha-\varepsilon)\tau_k}{\alpha(1-\tau_k)} - 1\right)/\partial\tau_k = \frac{1+\alpha-\varepsilon}{\alpha(1-\tau_k)^2} > 0$. From (41), $u^\star \in](\beta+\varrho)/B, 1[$ with $B > \beta + \varrho$. We have $\lim_{u\to(\beta+\varrho)/B} = -\infty$ and $\lim_{u\to 1} > 0$ because $B - \beta - \varrho > \beta(\beta+\varrho)$ (sufficient condition). Therefore, $u^\star \in](\beta+\varrho)/B, 1[$ is unique.

From the theorem of the implicit function we obtain:

$$u^\star = \mathcal{U}^K(\tau_k) \qquad \text{with} \qquad \mathcal{U}^K_{\tau_k}(\tau_k) < 0$$

[22]Note that $\frac{1-\tau_k}{\Upsilon(\tau_k)} = \frac{(1-\alpha)(1-\tau_k)}{1-\alpha+(\alpha-\varepsilon)\tau_k}$ and $\partial\frac{1-\tau_k}{\Upsilon(\tau_k)}/\partial\tau_k \leq 0$ (it comes directly from $\partial\frac{1-\tau_k}{\Upsilon(\tau_k)}/\partial\tau_k = -\frac{(1-\alpha)(1-\varepsilon)}{[1-\alpha+(\alpha-\varepsilon)\tau_k]^2} \leq 0$, $\forall(\alpha,\varepsilon) \in (0,1)$).

From (50), we have

$$l^\star = \mathcal{L}^K(\tau_k) \qquad \text{with} \;\; \mathcal{L}^K_{\tau_k}(\tau_k) < 0$$

and therefore

$$g^\star = B(1 - \mathcal{U}^K(\tau_k) - \mathcal{L}^K(\tau_k)) \qquad \text{with} \;\; g^\star_{\tau_k} > 0, \; \forall(\alpha, \varepsilon) \in]0, 1[$$